

Robust Runway Scheduling Using a Time-indexed Model

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Motivation

SESAR

- capacity: 3 fold increase
- safety: increase by a factor of 10
- environmental impact: 10% reduction by flights
- cost: 50% reduction

source: Concept Story Board 2009



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→ planning is necessary, but need of stable plans

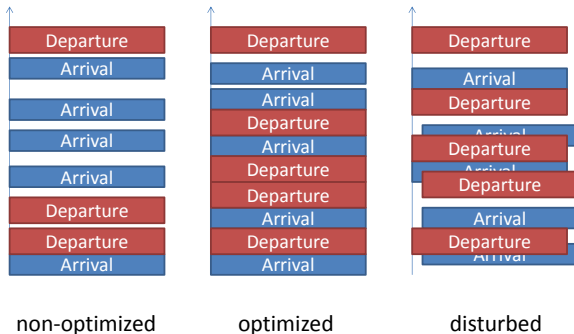


robust party preparation





Motivation



Robust optimization?



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Outline

Nominal Models

Robust Model

Simulation

Computational Results

Outlook



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Runway Scheduling

problem description

- given
 - single mixed-mode runway
 - set of aircraft with different weight classes
 - earliest, schedule and latest times for each aircraft
 - minimum separation times between two aircraft types
- task
 - schedule aircraft as close as possible to its schedule time
 - penalize, if scheduled time is later than latest time
 - fair schedules



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this is only an example!

Mathematical Model for Runway Scheduling (MIP) according to Beasley et al., Transportation Science, 2000

for each aircraft i determine target time t_i that solves:

$$\begin{array}{ll} \min & \sum_{i=1}^n \left((t_i - \tau_i^{ST})^2 + (t_i^{LT+})^2 \right) \\ \text{subject to} & \end{array}$$

$$\begin{array}{ll} t_i \geq \tau_i^{ET} & \forall i \in \{1, \dots, n\} \\ t_i - \tau_i^{LT} \leq t_i^{LT+} & \forall i \in \{1, \dots, n\} \end{array}$$

$$\begin{array}{ll} t_i + \delta_{i,j} - t_j \leq M_{i,j} \cdot (1 - m_{i,j}), & \forall i, j \in \{1, \dots, n\}, i < j \\ t_j + \delta_{j,i} - t_i \leq M_{j,i} \cdot m_{i,j}, & \forall i, j \in \{1, \dots, n\}, i < j \\ m_{i,j} \in \{0, 1\}, M_{i,j} \text{ "large" constant} & \end{array}$$

τ_i^{ST} : schedule time, τ_i^{ET} : earliest time,

τ_i^{LT} : latest time, $\delta_{i,j}$: separation time



setting for computation

- schedule times are chosen in steps of 5 minutes (schedules of airports) and 4 aircraft (arr/dep) per schedule time
- random data in earliest times depending on schedule times
- latest times are computed depending on earliest times
- weight class (H, M, L) and operation type (Arr, Dep) are uniformly distributed
- 10 runs with 20, 35 and 50 aircraft



MIP results

20 aircraft	MIP
<i>Runtime [s]</i>	2760
<i>Avg. deviation from schedule time [s]</i>	37.4
35 aircraft	
<i>Runtime [s]</i>	>3600
<i>Avg. deviation from schedule time [s]</i>	50.8
<i>Gap %</i>	81.6
50 aircraft	
<i>Runtime [s]</i>	> 3600
<i>Avg. deviation from schedule time [s]</i>	59.6
<i>Gap %</i>	93.6



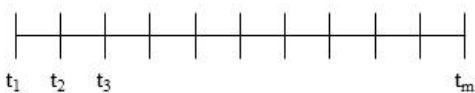
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alternatives?

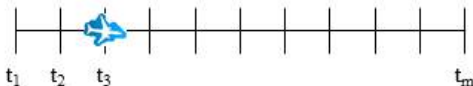
Time-indexed Model for Runway Scheduling (TIM)

- divide the time horizon into slots $T = \{t_1, t_1 + \Delta t, \dots, t_m\}$
- $T_i = T \cap [\tau_i^{ET}, \tau_i^{LT}]$



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- variables $b_{i,j} = \begin{cases} 1, & \text{if aircraft } i \text{ scheduled is scheduled on slot } t_j \\ 0, & \text{otherwise} \end{cases}$

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- variables $b_{i,j} = \begin{cases} 1, & \text{if aircraft } i \text{ scheduled is scheduled on slot } t_j \\ 0, & \text{otherwise} \end{cases}$
- separation times have to be considered



mathematical formulation:

according to Dyer/Wolsey, Discrete Applied Mathematics, 1990

$$\min \sum_{i=1}^n \sum_{j \in T_i} ((t_j - \tau_i^{ST})^2 + (t_{i,j}^{LT+})^2) \cdot b_{i,j}$$

subject to

each aircraft has to be scheduled

each slot can be used at most once

minimum separation time

mathematical formulation:

according to Dyer/Wolsey, Discrete Applied Mathematics, 1990

$$\min \sum_{i=1}^n \sum_{j \in T_i} ((t_j - \tau_i^{ST})^2 + (t_{i,j}^{LT+})^2) \cdot b_{i,j}$$

subject to

$$\sum_{j \in T_i} b_{i,j} = 1 \quad \forall i \in \{1, \dots, n\}$$

$$\sum_{i=1}^n b_{i,j} \leq 1 \quad \forall j \in T$$

$$b_{i,j} + \sum_{l=j+1}^{j + \lceil \frac{\delta_{i,k}}{\Delta t} \rceil} b_{k,l} \leq 1 \quad \forall i \in \{1, \dots, n\}, \forall j \in T_i, \forall k \neq i$$

$$b_{i,j} \in \{0, 1\}$$



MIP vs TIM

20 aircraft	MIP	TIM		
		$\Delta t = 5$	$\Delta t = 25$	$\Delta t = 75$
<i>Runtime [s]</i>	2760	2354	122	<1
<i>Avg. deviation from schedule time [s]</i>	37.4	41.5	48.2	83.1
35 aircraft				
<i>Runtime [s]</i>	>3600	>3600	112	<1
<i>Avg. deviation from schedule time [s]</i>	50.8	61.4	69.7	123.0
<i>Gap %</i>	81.6	16.6		
50 aircraft				
<i>Runtime [s]</i>	> 3600	>3600	252	<1
<i>Avg. deviation from schedule time [s]</i>	59.6	67.5	76.1	176
<i>Gap %</i>	93.6	29.8		

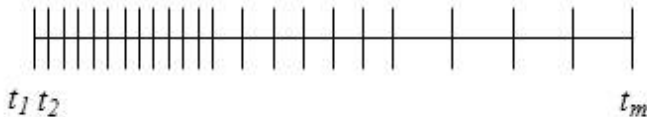


Dynamic Time-indexed Model for Runway Scheduling (DTIM)

idea

to be more precise the nearer an aircraft gets to touch-down/take-off

- different discretization for each aircraft possible
- time horizon individual for aircraft





MIP vs TIM/DTIM

20 aircraft	MIP	TIM			DTIM	
		$\Delta t = 5$	$\Delta t = 25$	$\Delta t = 75$	$\Delta t_1 = 5, \Delta t_2 = 75$	
					10acft	15acft
Runtime [s]	2760	2354	122	<1	2.3	8.5
Avg. deviation from schedule time [s]	37.4	41.5	48.2	83.1	51.1	49.9
35 aircraft						
Runtime [s]	>3600	>3600	112	<1	8.8	77.4
Avg. deviation from schedule time [s]	50.8	61.4	69.7	123.0	77.1	72.5
Gap %	81.6	16.6				
50 aircraft						
Runtime [s]	> 3600	>3600	252	<1	15.3	154.3
Avg. deviation from schedule time [s]	59.6	67.5	76.1	176	86.0	82.4
Gap %	93.6	29.8				



MIP vs TIM/DTIM

20 aircraft	MIP	TIM			DTIM	
		$\Delta t = 5$	$\Delta t = 25$	$\Delta t = 75$	$\Delta t_1 = 5, \Delta t_2 = 75$	
					10acft	15acft
Runtime [s]	2760	2354	122	<1	2.3	8.5
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50 aircraft						
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Avg. deviation from schedule time [s]	59.6	67.5	76.1	176	86.0	82.4
Gap %	93.6	29.8				

conclusion: consider TIM/DTIM further because of better performance when compared to MIP



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Goals

Robust Optimization For Runway Scheduling

- stable plans
- less replannings
- less go-arounds
- less departure slot losses

but first thought: robustness costs performance and throughput



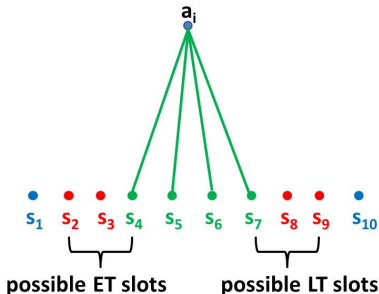
robustification

- uncertainties affect earliest and latest times
- robustification: immunize against deviations within a predefined uncertainty set

predefined uncertainty set/worst-case

- protect with high probability against
 - go-arounds
 - departure slot losses
 - replannings

Robustification of TIM/DTIM



- **strict robustness**
⇒ **delete all uncertain arcs**
- Bertsimas/Sim (Γ -approach)
 - Γ -many arcs can be uncertain for an aircraft
⇒ protection against the first Γ -many arcs
- recoverable and light robustness



Robust TIM/DTIM

engineering approach for strict robustness

- suppose every aircraft is disturbed by a normal distribution $N(\mu_i, \sigma_i^2)$
- take that time window, which is always feasible for the predefined uncertainty set: fewer slots can be assigned

$$T_i^R = \{t_1, t_1 + \Delta t_i, \dots, t_m\} \cap [\tau_i^{ET} + \mu_i + 2 \cdot \sigma_i, \tau_i^{LT} + \mu_i - 2 \cdot \sigma_i]$$

- minimum separation time constraint: increase buffer between two aircraft dependent on the expectation of aircraft i to be early/late



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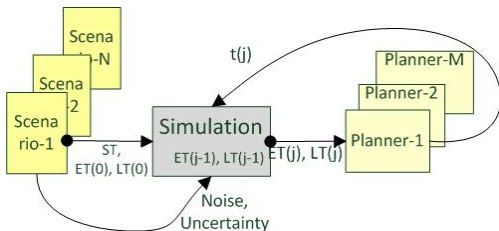
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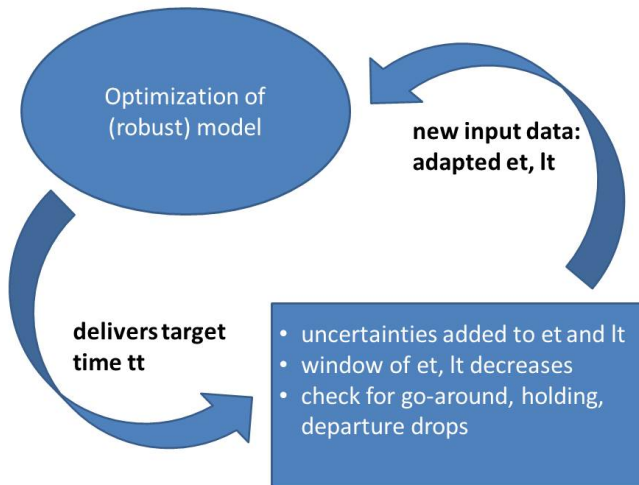
planner

- nominal planner: TIM
- robust planner 1: RobTIM
- robust planner 2: RobDTIM

scenarios

- scenario 1: high traffic, low uncertainty
- scenario 2: high traffic, high uncertainty
- scenario 3: medium traffic, high uncertainty

Simulation





test runs

- 3 scenarios
- 20 runs each
- 50 different randomly chosen aircraft per run
- high/med traffic (50 acft in 50mins/ 50acft 80 mins)
- high/low uncertainty

measurements

- go-arounds per simulation
- departure slot losses per simulation
- changed position per simulation
- changed target time per aircraft
- makespan
- objective function value/ avg. deviation from schedule time
- computational runtime



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Scenario 1: high traffic demand, low uncertainty

Table: Results scenario 1: RTIM vs. TIM (slot size 75s)

Measurements	RTIM	TIM	TIM - RTIM
GoAround	0.2	1.8	1.6
Dep. drop	0.15	0.4	0.25
Makespan [s]	3064	3144	80
			TIM/RTIM
Changed Pos / SimStep	0.29	0.69	2.38
Changed TT / acft [min]	1.95	3.24	1.67
Obj. func. value / acft [s]	276.5	359	1.29
Comp. runtime [s]	12.5	13.1	1.05



Scenario 1: high traffic demand, low uncertainty

Table: Results scenario 1: RDTIM (slot size 5s/75s) vs. RTIM (slot size 75s)

Measurements	RDTIM	RTIM	RTIM - RDTIM
Makespan [s]	2878	3064	186
			RTIM/RDTIM
Changed Pos / SimStep	0.91	0.29	0.31
Changed TT / acft [min]	3.34	1.95	0.58
Comp. runtime [s]	236.4	12.5	0.05



Scenario 2: high traffic demand, high uncertainty

Table: Results scenario 2: RTIM vs. TIM (slot size 75s)

Measurements	RTIM	TIM	TIM - RTIM
GoAround	0	2.4	2.4
Dep. drop	0	0.9	0.9
Makespan [s]	3269	3274	5
			TIM/RTIM
Changed Pos / SimStep	1.26	3.13	2.48
Changed TT / acft [min]	6.67	8.56	1.28
Obj. func. value / acft [s]	484.7	484.6	1.0
Comp. runtime [s]	31.8	26.4	0.83



Scenario 2: high traffic demand, high uncertainty

Table: RTIM for low and high uncertainty at high traffic

Measurements	lowU	highU	highU - lowU
GoAround	0.2	0	-0.2
Dep. drop	0.15	0	-0.15
Makespan [s]	3064	3269	205
			highU/lowU
Changed Pos / SimStep	0.29	1.26	4.34
Changed TT / acft [min]	1.95	6.67	3.42
Obj. func. value / acft [s]	276.5	484.7	1.75
Comp. runtime [s]	12.5	31.8	2.54



Scenario 3: medium traffic demand, high uncertainty

Table: Results scenario 3: RTIM vs. TIM (slot size 75s)

Measurements	RTIM	TIM	TIM - RTIM
GoAround	0	1.1	1.1
Dep. drop	0	0.2	0.2
Makespan [s]	3982	4015	33
			TIM/RTIM
Changed Pos / SimStep	0.31	0.62	2.0
Changed TT / acft [min]	2.5	4.86	1.94
Obj. func. value / acft [s]	96.9	157.2	1.62
Comp. runtime [s]	15.2	26.9	1.76



Scenario 3: medium traffic demand, high uncertainty

Table: RTIM for med./high traffic at high uncertainty

Measurements	medTr	highTr	highTr - medTr
GoAround	0	0	0
Dep. drop	0	0	0
Makespan [s]	3982	3269	-713
			highTr/medTr
Changed Pos / SimStep	0.31	1.26	4.06
Changed TT / acft [min]	2.5	6.67	2.68
Obj. func. value / acft [s]	96.9	484.7	5.0
Comp. runtime [s]	15.2	31.8	2.1



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Conclusion/Future work

- further research in context of dynamic time-indexed model
 - eliminate disadvantages in position/target time changes
- recoverable and light robustness approaches for the models
 - reduction of makespan
- choose more complex objective functions
 - reflect ATM operations in a more realistic way
 - we expect no significant effects on runtime efficiency



Conclusion/Future work

Goals

- stable plans ✓
- less replannings ✓
- less go-arounds ✓
- less departure slot losses ✓
- robustification is not costly ✓
- very fast ✓



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Thanks for your Robustness!